Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Bronze Level B1

Time: 1 hour 30 minutes

Materials required for examination papers	Items included with question
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
74	67	61	54	47	41

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - \mathrm{i} \text{ and } z_2 = -8 + 9\mathrm{i}$$

(*a*) Show z_1 and z_2 on a single Argand diagram.

(1)

Find, showing your working,

(*b*) the value of $|z_1|$,

(2)

(*c*) the value of arg z_1, giving your answer in radians to 2 decimal places,

(2)

(*d*) $\dfrac{z_2}{z_1}$ in the form $a + b\mathrm{i}$, where a and b are real.

(3)

June 2009

2.

$$z_1 = -2 + \mathrm{i}$$

(*a*) Find the modulus of z_1.

(1)

(*b*) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(*c*) Find z_2 and z_3, giving your answers in the form $p \pm \mathrm{i}\sqrt{q}$, where p and q are integers.

(3)

(*d*) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3 .

(2)

June 2011

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3. Given that $x = \frac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(*a*) the value of k,

(3)

(*b*) the other 2 roots of the equation.

(4)

June 2013

4.

$$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$$

(*a*) Use differentiation to find $f'(x)$.

(2)

The root α of the equation $f(x) = 0$ lies in the interval [0.7, 0.9].

(*b*) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

June 2011

5.

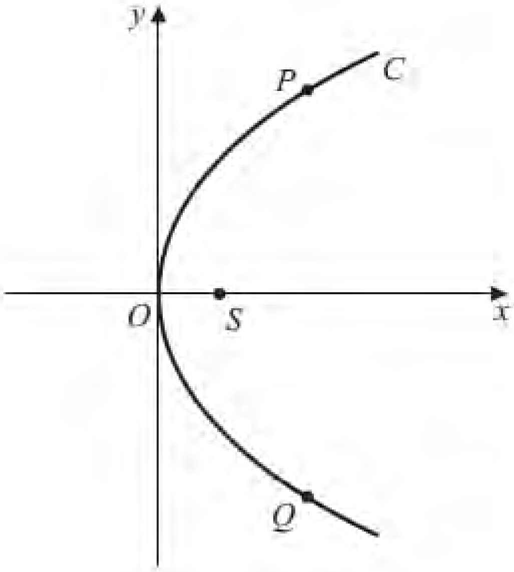

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$.
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(*a*) write down the y-coordinate of P,

(1)

(*b*) find the x-coordinate of P.

(2)

Figure 1 shows the point S which is the focus of C.

The line l passes through the point P and the point S.

(*c*) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

June 2012

6. Write down the 2×2 matrix that represents

(*a*) an enlargement with centre (0, 0) and scale factor 8,

(1)

(*b*) a reflection in the x-axis.

(1)

Hence, or otherwise,

(*c*) find the matrix **T** that represents an enlargement with centre (0, 0) and scale factor 8, followed by a reflection in the x-axis.

(2)

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

(*d*) Find **AB**.

(3)

Given that **AB** represents the same transformation as **T**,

(*e*) find the value of k and the value of c.

(2)

June 2010

7.

$$z = 2 - \mathrm{i}\sqrt{3}.$$

(*a*) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(*b*) $z + z^2$ in the form $a + b\mathrm{i}\sqrt{3}$, where a and b are integers,

(3)

(*c*) $\dfrac{z+7}{z-1}$ in the form $c + d\mathrm{i}\sqrt{3}$, where c and d are integers.

(4)

Given that

$$w = \lambda - 3\mathrm{i},$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(*d*) find the value of λ.

(2)

June 2012

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8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(a) Find the equation of the directrix of C.

(2)

(b) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0.$$

(4)

The tangent to C at the point (3, 12) meets the directrix of C at the point X.

(c) Find the coordinates of X.

(4)

June 2011

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9. The complex number w is given by

$$w = 10 - 5\text{i}$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places

(2)

The complex numbers z and w satisfy the equation

$$(2 + \text{i})(z + 3\text{i}) = w$$

(c) Use algebra to find z, giving your answer in the form $a + b\text{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\text{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

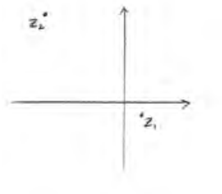
(d) find the value of λ.

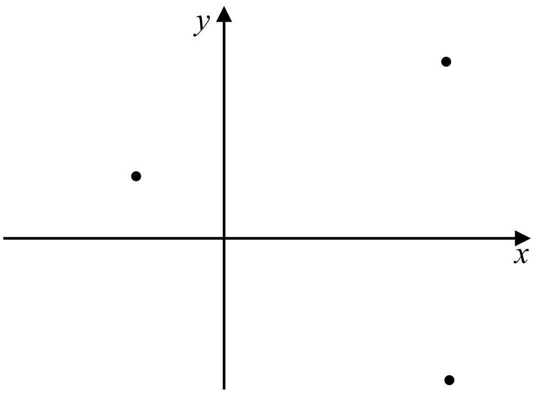
(2)

June 2013 (R)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)		B1
		(1)
(b)	$\lvert z_1 \rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1
		(2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$	M1
	$\arg z_1 = -0.46$ or 5.82 (awrt)	A1
		(2)
(d)	$\frac{-8+9\text{i}}{2-\text{i}} \times \frac{2+\text{i}}{2+\text{i}}$	M1
	$= \frac{-16-8\text{i}+18\text{i}-9}{5} = -5+2\text{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	A1 A1ft
		(3)
		[8]

Question Number	Scheme	Marks
2. (a)	$\lvert z_1 \rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	B1
		(1)
(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	M1
	$= 2.677945045... = 2.68$ (2 dp)	A1 oe
		(2)
(c)	$z^2 - 10z + 28 = 0$	
	$z = \dfrac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	M1
	$= \dfrac{10 \pm \sqrt{100 - 112}}{2}$	
	$= \dfrac{10 \pm \sqrt{-12}}{2}$	
	$= \dfrac{10 \pm 2\sqrt{3}\,\mathrm{i}}{2}$	M1
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\ q = 3\}$	A1 oe
		(3)
(d)	 The points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$.	
	The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label.	B1
	The distinct points z_2 and z_3 plotted correctly and symmetrically about the x-axis on the Argand diagram with/without label.	B1 $\sqrt{}$
		(2)
		[8]

Question Number	Scheme	Marks
3. (a)	**Ignore part labels and mark part (a) and part (b) together.**	
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-9\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-13$	M1
	$\left(\frac{1}{4}\right)-\left(\frac{9}{4}\right)+\left(\frac{k}{2}\right)-13=0 \Rightarrow k=......$	dM1
	$k = 30$	A1 cao (3)
(b)	$f(x)=(2x-1)(x^2-4x+13)$ or $\left(x-\frac{1}{2}\right)(2x^2-8x+26)$	M1
	$x^2-4x+13$ or $2x^2-8x+26$	A1
	$x=\frac{4\pm\sqrt{4^2-4\times13}}{2}$ or equivalent	M1
	$x=\frac{4\pm6i}{2}=2\pm3i$	A1 oe
		(4) **[7]**
4. (a)	$f(x)=x^2+\frac{5}{2x}-3x-1,\quad x\neq0$	
	$f(x)=x^2+\frac{5}{2}x^{-1}-3x-1$	
	$f'(x)=2x-\frac{5}{2}x^{-2}-3\ \{+0\}$	M1
	$\left\{f'(x)=2x-\frac{5}{2x^2}-3\right\}$	A1
		(2)
(b)	$f(0.8)=0.8^2+\frac{5}{2(0.8)}-3(0.8)-1(=0.365)\left(=\frac{73}{200}\right)$	B1
	$f'(0.8)=-5.30625\left(=\frac{-849}{160}\right)$	M1
	$\alpha_2 = 0.8-\left(\frac{"0.365"}{"-5.30625"}\right)$	M1
	$= 0868786808...$	
	$= 0.869$ (3dp)	A1 cao
		(4) **[6]**

Question Number	Scheme	Marks
5. (a)	$C: y^2 = 8x \Rightarrow a = \frac{8}{4} = 2$	
	$PQ = 12 \Rightarrow$ By symmetry $y_P = \frac{12}{2} = \underline{6}$	B1
		(1)
(b)	$y^2 = 8x \Rightarrow 6^2 = 8x$	M1
	$\Rightarrow x = \frac{36}{8} = \frac{9}{2}$ (So P has coordinates $\left(\frac{9}{2}, 6\right)$)	A1 oe
		(2)
(c)	Focus $S(2, 0)$	B1
	Gradient $PS = \dfrac{6-0}{\frac{9}{2}-2}$ $\left\{= \dfrac{6}{\left(\frac{5}{2}\right)} = \dfrac{12}{5}\right\}$	M1
	Either $y - 0 = \frac{12}{5}(x-2)$ or $y - 6 = \frac{12}{5}(x - \frac{9}{2})$;	
	or $y = \frac{12}{5}x + c$ and $0 = \frac{12}{5}(2) + c \Rightarrow c = -\frac{24}{5}$;	M1
	l: $\underline{12x - 5y - 24 = 0}$	A1
		(4)
		[7]
6. (a)	$\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$	B1
		(1)
(b)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$	M1 A1
		(2)
(d)	$\mathbf{AB} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6k+c & 0 \\ 4k+2c & -8 \end{pmatrix}$	M1 A1 A1
		(3)
(e)	"$6k + c = 8$" and "$4k + 2c = 0$"	M1
	$k = 2$ and $c = -4$	A1
		(2)
		[9]

Question Number	Scheme	Marks
7. (a)	$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	M1
	$= -0.7137243789.. = -0.71$ (2 dp)	A1
		(2)
(b)	$z^2 = \left(2 - \mathrm{i}\sqrt{3}\right)\left(2 - \mathrm{i}\sqrt{3}\right)$	M1
	$= 4 - 2\mathrm{i}\sqrt{3} - 2\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	
	$= 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} - 3\right)$	
	$= 2 - \mathrm{i}\sqrt{3} + \left(1 - 4\mathrm{i}\sqrt{3}\right)$	M1A1
	$= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	
		(3)
(c)	$\frac{z+7}{z-1} = \frac{2 - \mathrm{i}\sqrt{3} + 7}{2 - \mathrm{i}\sqrt{3} - 1}$	M1
	$= \frac{\left(9 - \mathrm{i}\sqrt{3}\right)}{\left(1 - \mathrm{i}\sqrt{3}\right)} \times \frac{\left(1 + \mathrm{i}\sqrt{3}\right)}{\left(1 + \mathrm{i}\sqrt{3}\right)}$	**d**M1
	$= \frac{9 + 9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3} + 3}{1 + 3}$	
	$= \frac{12 + 8\mathrm{i}\sqrt{3}}{4}$	M1
	$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3, d = 2$.)	A1
		(4)
(d)	$w = \lambda - 3\mathrm{i}$, and $\arg(4 - 5\mathrm{i} + 3w) = -\frac{\pi}{2}$	
	$\left(4 - 5i + 3w = 4 + 3\lambda - 14\mathrm{i}\right)$	
	So real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	M1
	So, $\lambda = -\frac{4}{3}$	A1
		(2)
		[11]

Question Number	Scheme	Marks
8. (a)	$C: y^2 = 48x$ with general point $P(12t^2, 24t)$.	
	$y^2 = 4ax \Rightarrow a = \frac{48}{4} = 12$	M1
	So, directrix has the equation $x + 12 = 0$	A1 oe
		(2)
(b)	$y = \sqrt{48}\, x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\sqrt{48}\, x^{-\frac{1}{2}} \left(= 2\sqrt{3}\, x^{-\frac{1}{2}}\right)$	
	or (implicitly) $y^2 = 48x \Rightarrow 2y\frac{dy}{dx} = 48$	M1
	or (chain rule) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 24 \times \frac{1}{24t}$	
	When $x = 12t^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{48}}{2\sqrt{12t^2}} = \frac{\sqrt{4}}{2t} = \frac{1}{t}$	
		A1
	or $\frac{dy}{dx} = \frac{48}{2y} = \frac{48}{48t} = \frac{1}{t}$	
	T: $y - 24t = \frac{1}{t}\left(x - 12t^2\right)$	M1
	T: $ty - 24t^2 = x - 12t^2$	
	T: $x - ty + 12t^2 = 0$	A1 cso*
		(4)
(c)	Compare $P(12t^2, 24t)$ with $(3, 12)$ gives $t = \frac{1}{2}$.	B1
	$t = \frac{1}{2}$ into **T** gives $x - \frac{1}{2}y + 3 = 0$	M1
	At X, $x = -12 \Rightarrow -12 - \frac{1}{2}y + 3 = 0$	M1
	So, $-9 = \frac{1}{2}y \Rightarrow y = -18$	
	So the coordinates of X are $(-12, -18)$.	A1
		(4)
		[10]

Question Number	Scheme	Marks
9. (a)	$w = 10 - 5\mathrm{i}$	
	$\lvert w \rvert = \left\{\sqrt{10^2 + (-5)^2}\right\} = \sqrt{125}$ or $5\sqrt{5}$ or 11.1803...	B1
		(1)
(b)	$\arg w = -\tan^{-1}\left(\frac{5}{10}\right)$	M1
	$= -0.463647609... = -0.46$ (2 dp)	A1 oe
		(2)
(c)	$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$	
	$z + 3\mathrm{i} = \dfrac{10 - 5\mathrm{i}}{(2 + \mathrm{i})}$	B1
	$z + 3\mathrm{i} = \dfrac{(10 - 5\mathrm{i})}{(2 + \mathrm{i})} \times \dfrac{(2 - \mathrm{i})}{(2 - \mathrm{i})}$	M1
	$z + 3\mathrm{i} = \dfrac{20 - 10\mathrm{i} - 10\mathrm{i} - 5}{1 + 4}$	M1
	$z + 3\mathrm{i} = \dfrac{15 - 20\mathrm{i}}{5}$	
	$z + 3\mathrm{i} = 3 - 4\mathrm{i}$	
	$z = 3 - 7\mathrm{i}$ (Note: $a = 3$, $b = -7$.)	A1
		(4)
(d)	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4}$	
	$\lambda + 9\mathrm{i} + w = \lambda + 9\mathrm{i} + 10 - 5\mathrm{i} = (\lambda + 10) + 4\mathrm{i}$	
	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4} \Rightarrow \lambda + 10 = 4$	M1
	So, $\lambda = -6$	A1
		(2)
		[9]

Examiner reports

Question 1

Almost all candidates achieved the mark in part (a) for the argand diagram. Also the modulus of a complex number was understood and usually found correctly. Finding the argument of the complex number caused more problems for some candidates as a number of them did not consider which quadrant they needed. Also some candidates used incorrect trigonometry. A few answers were given in degrees and some calculated tan (0.5) instead of arctan (0.5). Part (d), which involved calculating a quotient, was usually answered correctly also. Most successfully multiplied by the conjugate 2 – i and got full marks. Some misread this part and found $\frac{z_1}{z_2}$ which meant that they could only get a maximum of 1 mark for multiplying by their conjugate.

Question 2

Part (a) was well answered with most candidates gaining the mark. There were a few instances of $(-2)^2 + (\text{i})^2 = 4 - 1 = 3$.

In part (b) many could obtain the correct argument but many found arctan (0.5) = 0.46 or arctan (–0.5) = –0.46 and stopped. Candidates should be encouraged to consider where the point is on the Argand Diagram and use appropriate trigonometry.

Part (c) was also generally answered correctly with the most common approach being the use of the quadratic formula. Completing the square was relatively common and there were some approaches that used the sum and product of roots. Plotting the points on an Argand Diagram was usually good however there were a significant number of cases where the conjugate pair were incorrectly positioned relative to –2 + i.

Question 3

A variety of methods were used to find k including evaluating f(0.5), long division and inspection. Generally all were quite successful, but those who attempted long division or tried to find the quadratic factor by inspection often made errors. A small number of candidates substituted $x = -\frac{1}{2}$ instead of $x = \frac{1}{2}$ and gained no marks in part (a).

Once $k = 30$ had been successfully achieved, part (b) proved very accessible for most candidates. Once again, the quadratic factor was obtained from a variety of methods, with long division being the most popular but there were often a few sign errors here. Once the quadratic factor had been obtained, candidates went on to find the two complex solutions either by completing the square or by using the formula.

Question 4

In part (a) the differentiation was often successful however a significant number of candidates had difficulty dealing with the “2” in the denominator of the second term. Before differentiating, the term was often seen to be written correctly as $5(2x)^{-1}$ but this then caused problems because of the chain rule requirement.

In part (b), the Newton-Raphson process was clearly well rehearsed and many could apply it correctly. However, as with question 1, candidates should be encouraged to show all their working e.g. showing clearly their evaluation of f(0.8) and f ′(0.8).

Question 5

Parts (a) and (b) were usually very well answered although there was some confusion with coordinates at times with $x = 6$ being substituted into the equation for the parabola rather than $y = 6$. In part (c) the majority of candidates correctly identified the focus and followed this with correct work to find the equation of the line *PS*. Some candidates thought that a tangent was involved and proceeded to differentiate the equation for the parabola in order to establish the gradient, with no reference to the focus. It was disappointing to see a significant number of candidates failing to comply with the demand to have integer coefficients for the straight line.

Question 6

Part (a) was generally well done. Some however did have the columns of their matrix the wrong way round. Part (b) was similar to (a), but slightly less well done, with both sign and column errors. Candidates who have been taught to sketch a graph and transform a unit triangle/square performed well throughout this question.

There were a noticeable minority who appeared to have no idea about using matrices for transformations, which meant a loss of access to some fairly straightforward marks. Many candidates multiplied their matrices in the wrong order for part (c). In this particular case it made no difference, and was not penalised, but it suggests that many candidates were not aware of the correct order.

The majority of candidates knew how to multiply matrices in part (d) and were able to achieve a correct answer. In part (e), most candidates were successful in obtaining the correct equations. Of those who didn’t, the majority were able to follow through from their (c) and (d). As in many other questions, candidates should be encouraged to check that their final solutions are consistent with their matrices.

Question 7

In part (a) the majority of candidates could at least calculate the magnitude of the angle concerned but there were many cases where the sign was omitted. There were very few cases where candidates worked in degrees.

In attempting $z + z^2$ in part (b) most candidates made sound attempts at z^2 although there were some instances of poor algebra and sometimes insufficient work to show that $i^2 = -1$ but on the whole, the correct answer was seen very frequently.

In part (c) almost all candidates could substitute correctly for z and also the method for making the denominator real was well known. There were again some cases of poor algebra and/or an inability to deal with directed numbers correctly.

Part (c) proved to be a good discriminator and it was often the case that either the candidate knew what to do straight away or spent quite a lot of time making little progress.

Question 8

In part (a) most candidates gave the correct equation for the directrix although $x = 12$ was seen occasionally.

In part (b) all three methods for establishing the gradient were seen but with direct differentiation, after taking the square root, being the most common. Those with a correct gradient almost always proceeded to establish the correct equation for the tangent. A significant number of candidates quoted the gradient with no evidence of any calculus and these lost marks as the answer was given and this was a “show that” question.

There were a variety of approaches to part (c). The majority started with identifying the value of t and substituted this with their value of x from part (a) to correctly identify the coordinates of the point X.

Question 9

Part (a) was usually answered correctly and in part (b) the argument was generally correct, but a minority gave the positive argument by mistake. In part (c) a variety of methods were used to find z. Most candidates were successful, but some had no clear method to account for the two complex numbers when solving. In part (d) many candidates found the value of λ correctly, most of these through the use of trigonometry rather than by equating real and imaginary parts directly.

Statistics for FP1 Practice Paper Bronze Level B1

				Mean score for students achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	8		90	7.16		7.73	7.36	6.99	6.41	5.61	4.68
2	8		90	7.18	7.91	7.73	7.34	6.99	6.52	6.10	4.77
3	7	7	90	6.31	6.95	6.76	6.43	6.08	5.90	5.48	4.43
4	6		92	5.51	5.99	5.85	5.64	5.39	5.13	4.84	4.03
5	7		89	6.20	6.89	6.76	6.43	6.09	5.68	4.86	4.10
6	9		89	8.01	8.90	8.74	8.35	7.75	7.22	6.43	4.41
7	11		80	8.84	10.60	9.88	8.87	8.33	7.65	7.01	5.69
8	10		83	8.28	9.96	9.60	8.68	7.66	6.35	5.04	3.17
9	9		83	7.49	8.76	8.33	7.85	6.92	6.20	6.36	4.63
	75		**87**	**64.98**		**71.38**	**66.95**	**62.20**	**57.06**	**51.73**	**39.91**